

Final Exam - Statistics IV
B. Math 3rd Year, Second Semester

19 May, 2021

- (i) Total number of points for Part I (Theory) of the final exam is 90.
- (ii) Duration of Part I is 3 hours.
- (iii) You may consult the tables for t -distribution, F -distribution and z -distribution attached with the question paper.

Name: _____

Roll Number: _____

Part I - Theory

1. Let $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i, 1 \leq i \leq n$, denote the normal error simple linear regression model. Here X_i denotes the value of the predictor variable in the i th trial, Y_i denotes the value of the response variable in the i th trial, β_0 and β_1 are parameters. Let \bar{X}, \bar{Y} denote the mean of the observed values of the predictor variable, response variable, respectively.
 - (a) (3 points) State the assumptions regarding the error terms ε_i .
 - (b) (5 points) Show that the point (\bar{X}, \bar{Y}) lies on the fitted regression line.
 - (c) (10 points) Recall that the SSR is defined as $\sum_{i=1}^n (\hat{Y}_i - Y_i)^2$, where \hat{Y}_i denotes the fitted value corresponding to X_i . Show that

$$SSR = b_1^2 \sum_{i=1}^n (X_i - \bar{X})^2,$$

where b_1 denotes the least-squares estimator for β_1 .

- (d) (12 points) Compute the variance of b_1 and show that the variability in b_1 is decreased if the values of X are spread out.

2. Consider the normal error multiple regression model:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \cdots + \beta_{p-1} X_{i,p-1} + \varepsilon_i, 1 \leq i \leq n.$$

- (a) (10 points) Show that the normal equation related to minimizing the penalty function:

$$Q(\vec{\beta}) = \sum_{i=1}^n (Y_i - \mathbf{X}_i^T \vec{\beta})^2 + c \left(\sum_{i=1}^{p-1} \beta_i^2 \right), \quad (1)$$

is given by,

$$(\mathbf{X}^T \mathbf{X} + cI_p) \vec{\beta} = \mathbf{X}^T \vec{Y}.$$

- (b) (5 points) The parameter c is called the *regularization parameter*. Note that the usual linear regression model corresponds to the value of $c = 0$. To remedy what problem in multiple linear regression is the ridge regression model employed? What happens to the bias and variance of the estimators of the regression parameters when the parameter c is set at a very high value?

Total for Question 2: 15

3. The sales manager of a company wishes to investigate how sales performance, Y , depends on five independent predictor variables:

- (i) $X_1 \rightarrow$ number of months the sales representative has been employed by the company;
- (ii) $X_2 \rightarrow$ sales of the company's product and competing products in the sales territory;
- (iii) $X_3 \rightarrow$ advertising expenditure in the territory;
- (iv) $X_4 \rightarrow$ weighted average of the company's market share in the territory for the previous four years;
- (v) $X_5 \rightarrow$ change in the company's market share in the territory over the previous four years

A random sample of 26 observations shows the following results:

Variable	Coefficient	Standard Error
Intercept	-1050.00	420.00
X_1	3.60	1.20
X_2	0.04	0.01
X_3	0.13	0.04
X_4	250.00	175.00
X_5	350.00	200.00

Partial ANOVA table:

Source of variation	SS
SSR (Regression)	39500
SSE (Residuals)	3500
SSTO (Total)	43000

- (a) (3 points) Complete the ANOVA table with the information about degrees of freedom, and mean squared values.
- (b) (5 points) Using a 5% significance level, test whether there is a regression relation between Y and the predictor variables. (Clearly state the hypotheses, decision rule, test statistic, and summarize your conclusion.)
- (c) (7 points) Perform statistical tests to identify the predictor variables that are significant in predicting sales performance. (Clearly state the hypotheses, decision rule, test statistic, and summarize your conclusion.)
- (d) (10 points) If a simple regression analysis is performed with X_1 as the only predictor variable, it is observed that the SSE is 3 times as large as the SSE in the multiple regression model with 5 predictor variables. Test whether the simple regression model is significant or not. (Clearly state the hypotheses, decision rule, test statistic, and summarize your conclusion.)

Total for Question 3: 25

4. Sudden death is an important, lethal cardiovascular endpoint. Most previous studies of risk factors for sudden death have focused on men. Looking at this issue for women is important as well. For this purpose, data were used from the Framingham Heart Study. Several potential risk factors, such as age, blood pressure and cigarette smoking are of interest and need to be controlled for simultaneously. Therefore a multiple logistic regression was fitted to these data shown below. The response variable is 2-year incidence of sudden death in females without prior coronary heart disease.

Risk Factor	Regression Coefficient	Standard Error
Intercept term	-15.3	0.1
Blood Pressure (mm Hg)	0.0019	0.0070
Weight (% of study mean)	-0.0060	0.0100
Cholesterol (mg/100 mL)	0.0056	0.0029
Glucose (mg/100 mL)	0.0066	0.0038
Smoking (cigarettes/day)	0.0069	0.0199
Hematocrit (%)	0.111	0.049
Vital capacity (centilitres)	-0.0098	0.0036
Age (years)	0.0686	0.0225

- (a) (4 points) Which of the above eight risk factors are statistically significant? (Clearly state the hypotheses, decision rule, test statistic, and summarize your conclusion.)
- (b) (6 points) Compute the odds ratios relating the additional risk of sudden death associated with (i) a 10-centiliter decrease in vital capacity and (ii) an additional

year of age, after adjusting for the other risk factors. Give brief interpretations of the age and vital capacity coefficients. (You may use the approximation $e^x \approx 1 + x$ for small x .)

- (c) (4 points) Provide 95% confidence intervals for the odds ratios in part (b).
- (d) (6 points) Based on the risk factors retained in the model after your analysis in part (a), predict the probability of sudden death for a 50 year old woman with systolic blood pressure of 120 mm Hg, a relative weight of 100 %, a cholesterol level of 250 mg/100mL, a glucose level of 100 mg/100mL, a hematocrit of 40 %, and a vital capacity of 450 centiliters who smokes 10 cigarettes per day.

Total for Question 4: 20

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
Z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

95th percentile values for the F -distribution

$\nu_1 \rightarrow$ degrees of freedom in numerator.

$\nu_2 \rightarrow$ degrees of freedom in denominator.

$\nu_1 \backslash \nu_2$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Source: E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians*, Vol. 2 (1972), Table 5, page 178, by permission.

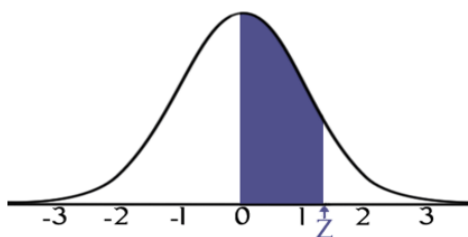
99th percentile values for the F -distribution

$\nu_1 \rightarrow$ degrees of freedom in numerator.

$\nu_2 \rightarrow$ degrees of freedom in denominator.

$\nu_1 \backslash \nu_2$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4052	5000	5403	5625	5764	5859	5928	5981	6023	6056	6106	6157	6209	6235	6261	6287	6313	6339	6366
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	26.9	26.7	26.6	26.5	26.4	26.3	26.2	26.1
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.7	13.6	13.5
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.82	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

Source: E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians*, Vol. 2 (1972), Table 5, page 180, by permission.



STANDARD NORMAL TABLE (Z)

Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean (0) and z is 0.3944.

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

Final Exam - Statistics IV
B. Math 3rd Year, Second Semester

19 May, 2021

Name: _____

Roll Number: _____

Part II - Practical

- (i) Total number of points for Part II (Practical) of the final exam is 30.
- (ii) Deadline for completion of Part II is 24 hours after start of final exam.
- (iii) You may use R or Python to solve the problems.
- (iii) Source code with comments and write-up regarding justification of the conclusions must be submitted for grading.

1. (a) (4 points) Generate 1000 pairs of random vectors from the unit sphere in \mathbf{R}^{10} (uniformly with respect to the surface measure of the sphere) and compute the correlation between the vectors in each case. Draw a histogram of the correlation values.
- (b) (6 points) In a normal error simple linear regression model, the observed coefficient of correlation (between the observed response vector \vec{Y} and fitted response vector $\vec{\hat{Y}}$) for a given dataset of 10 pairs of observed (X_i, Y_i) values, is 0.10. At the 5% significance level, formulate a hypothesis test and state the decision for whether the correlation is significantly different from 0 or not. Provide justification for the steps.

Total for Question 1: 10

2. The shared dataset provides selected country demographic information (CDI) for 440 of the most populous counties in the United States. Each line of the dataset has an identification number with a county name and state abbreviation and provides information on 14 variables for a single county. The 17 variables are given in the table below. Our goal is to predict the total number of active physicians (Y) in a CDI.

- (a) (6 points) There are two alternative models under consideration.
 Model I includes the following predictor variables: Total population (X_1), Land area (X_2), and total personal income (X_3).
 Model II includes the following predictor variables: Population density (total population divided by land area, X_1), percent of population 65 or older (X_2), total personal income (X_3).
 For each proposed model, fit the first-order regression model. Calculate R^2 for each model. Is one model clearly preferable than the other in terms of this measure?
- (b) (14 points) It has been decided to include total population (X_1) and total personal income (X_2) as predictor variables. We want to decide whether to include an additional suitable predictor variable.
- (i) (4 points) For each of the following variables, calculate the coefficient of partial determination given that X_1 and X_2 are included in the model: land area (X_3), percent of population 65 or older (X_4), number of hospital beds (X_5), and total serious crimes (X_6).
- (ii) (4 points) On the basis of results in part (i), which of the four additional predictor variables is best? Is extra sum of squares associated with this variable larger than those for the other three variables?
- (iii) (6 points) Test whether or not the variable determined to be best in part (ii) is helpful in the regression model when X_1 and X_2 are included in the model: Use a significance level of 0.01.

Variable Number	Description
1	Identification number (1 – 440)
2	County name
3	State (Two-letter abbreviation)
4	Land area (in square miles)
5	Total population
6	Percent of population aged 18 – 34
7	Percent of population 65 or older
8	Number of active physicians
9	Number of hospital beds
10	Total serious crimes
11	Percent high school graduates
12	Percent bachelor's degrees
13	Percent below poverty level
14	Percent unemployment
15	Per capita income (in dollars)
16	Total personal income (in millions of dollars)
17	Geographic region (1 = NE, 2 = NC, 3 = S, 4=W)

Total for Question 2: 20

Sheet1

1 Los_Angeles CA	4060	8863164	32.1	9.7	23677	27700	688936	70	22.3	11.6	8	20786	184230	4
2 Cook IL	946	5105067	29.2	12.4	15153	21550	436936	73.4	22.8	11.1	7.2	21729	110928	2
3 Harris TX	1729	2818199	31.3	7.1	7553	12449	253526	74.9	25.4	12.5	5.7	19517	55003	3
4 San_Diego CA	4205	2498016	33.5	10.9	5905	6179	173821	81.9	25.3	8.1	6.1	19588	48931	4
5 Orange CA	790	2410555	32.6	9.2	6062	6369	144524	81.2	27.8	5.2	4.8	24400	58818	4
6 Kings NY	71	2300664	28.3	12.4	4861	8956	680966	63.7	16.6	19.5	9.5	16803	38558	1
7 Manicopa AZ	9204	2122101	29.2	12.5	4320	6104	177593	81.5	22.1	19.8	4.9	18042	38587	4
8 Wayne MI	614	2111687	27.4	12.5	3823	9490	193978	70	13.7	16.9	10	17461	36872	2
9 Dade FL	1945	1937094	27.1	13.9	6274	8840	244725	65	18.8	14.2	8.7	17823	34525	3
10 Dallas TX	880	1852810	32.6	8.2	4718	6934	214258	77.1	26.3	10.4	6.1	21001	38911	3
11 Philadelphia PA	135	1585577	29.1	15.2	6641	10494	109148	64.3	15.2	16.1	8	16721	26512	1
12 King WA	2126	1507319	31.1	11.1	5280	4009	124959	88.2	32.8	5	4.6	23779	35843	4
13 Santa_Clara CA	1291	1497577	32.6	8.7	4101	3342	77009	82	32.6	5	5.5	25193	37728	4
14 San_BernardCA	20062	1418380	30.1	8.8	2463	3349	83110	75.4	14.9	10.3	8	16399	23260	4
15 Cuyahoga OH	458	1412140	26.3	15.6	5620	8132	73150	74	20.1	11	5.5	21086	29776	2
16 Middlesex MA	824	1398468	31.7	12.5	5158	4152	35825	84.3	35.4	4.2	7.3	25312	35398	1
17 Allegheny PA	730	1336449	26.2	17.4	5281	8436	50186	79	22.6	8.7	5.3	20681	27639	1
18 Suffolk NY	911	1321864	27.9	10.8	3021	3904	66723	82.2	23	3	7	24262	32071	1
19 Nassau NY	287	1287348	25.7	14.2	6147	5200	43203	84.2	30	2.5	5.1	31679	40782	1
20 Alameda CA	738	1279182	30.8	10.6	3169	3284	107338	81.4	28.8	8	5.3	22148	28531	4
21 Broward FL	1209	1255488	25.3	20.7	2456	5543	107386	76.8	18.8	7.1	7.4	22355	28066	3
22 Bexar TX	1247	1185394	29.5	9.9	3062	4086	133098	72.7	19.7	16.2	6.7	15508	18383	3
23 Riverside CA	7208	1170413	27.9	13.2	1385	2435	95494	74.1	14.6	10.7	11.7	17185	20114	4
24 Tarrant TX	864	1170103	32.2	8.3	1677	3672	132495	79.9	24	8.2	6.6	18825	22027	3
25 Oakland MI	873	1083592	27.6	10.9	4020	3254	50964	64.6	30.2	4.4	7.3	26884	29131	2
26 Sacramento CA	966	1041219	29.7	10.6	2464	2855	84305	82.2	23	9.8	6.3	18934	19174	4
27 Hennepin MN	557	1032431	31.6	11.3	3065	5395	71753	88.2	31.6	6.4	4.3	23705	24474	2
28 St_Louis MO	508	993529	26.1	13.1	1194	1056	42595	82.3	29.2	4	5.1	24219	24062	2
29 Erie NY	1045	968532	27.3	15.2	2748	4632	55306	76.4	20	9.4	6.8	18305	17729	1
30 Franklin OH	540	961437	33.5	9.6	2675	4011	82680	81	26.6	9.1	4.2	19040	18306	2
31 Milwaukee WI	242	959275	29.3	13.6	2774	4141	73681	76.3	19.3	12.6	4.9	18431	17680	2
32 Westchester NY	433	874866	26.3	14.4	4577	3540	37118	81	35.3	4.7	5.4	33330	29159	1
33 Hamilton OH	407	866228	28	13.3	3164	4683	57208	75.6	23.7	10.3	4.5	20580	17827	2
34 Palm_Beach FL	1974	863518	23.3	24.4	1833	3164	76142	78.8	22.1	6.2	8.4	26798	23141	3
35 Hartford CT	736	851783	28.3	14.1	2851	2940	51926	77.7	25.8	6	6.9	24875	21188	1
36 Pinellas FL	280	851659	22.6	26	1620	4458	62344	78.1	18.5	6.2	6.2	21610	18404	3
37 Honolulu HI	600	836231	30.4	11	2025	2174	51032	81.2	24.6	5.4	2.3	21307	17818	4
38 Hillsborough FL	1051	834054	29.4	12.2	2012	3049	89895	75.6	20.2	9.5	6	16876	14075	3
39 Fairfield CA	827645	26.7	13.3	2417	4948	44374	81	34.2	4.5	5.9	32342	26769	1	
40 Shelby TN	755	826330	29.4	10.4	2489	4918	67032	75.1	20.8	14.7	5.4	18430	15229	3
41 Bergen NJ	234	825380	25.4	15.3	3226	2279	28521	81.6	31.7	2.7	5.2	32230	26602	1
42 Fairfax_CountVA	396	818584	29.2	6.5	1694	135	30202	91.4	49	2.2	3.2	28999	23738	3
43 New_Haven CT	606	804219	28.7	14.7	3161	2486	52903	77.5	24.2	6	7.3	22197	17851	1
44 Contra_Costa CA	720	803732	26.5	10.9	1761	1781	51243	86.5	31.6	5.5	5.6	25523	20514	4
45 Marion IN	396	797159	30.6	11.7	2936	4654	61004	76.8	21.4	9.3	5	19148	15264	2
46 DuPage IL	334	781666	29	8.7	2157	1842	29708	78.6	36	1.7	4.8	26772	20927	2
47 Essex NJ	126	778206	28.6	12.7	2811	4841	75595	70.1	24	11.3	7.9	24523	19084	1
48 Montgomery MD	495	757027	28.6	10.2	4635	1507	34754	90.6	49.9	2.7	3.3	30081	22772	3
49 Clark NV	7911	741459	29	10.5	969	2011	52786	77.3	13.8	7.5	5.8	18625	13810	4
50 Baltimore_CityMD	81	736014	30	13.7	5444	6203	87355	60.7	15.5	17.8	9.4	17263	12706	3
51 Prince_GeorgMD	486	729268	33.7	6.9	1253	1322	54469	83.2	25.5	3.7	5	19568	14270	3
52 Salt_Lake UT	737	725956	27.8	8.5	2094	2076	58610	85.3	23.8	7.7	4.5	15399	11179	4
53 San_Francisco CA	47	723959	32.2	14.5	4761	3840	71234	78	35	9.7	5.6	28532	20556	4
54 Macomb MI	480	717400	28.2	12.3	705	1202	41048	76.9	13.5	4	9.4	20924	15011	2
55 Monroe NY	659	713968	29	12.5	2438	3077	43780	80.1	26.3	7.7	4.4	21641	15451	1
56 Worcester MA	1513	709705	29.2	13.7	1902	2205	7099	77.4	22.2	6.3	10.2	19895	14120	1
57 Baltimore MD	599	692134	27.8	14	1269	641	46789	78.4	25	3.8	5.7	23470	16244	3
58 Montgomery PA	483	678111	26.1	15	3237	2425	20335	83.8	32.1	2.2	5	28462	19300	1
59 Orange FL	908	677491	33.2	10.6	1367	2929	52577	78.8	21.2	7.8	6.6	17879	12113	3
60 Duval FL	774	672971	30.7	10.7	1538	2623	68586	76.9	18.4	9.8	6.6	17662	11886	3
61 Middlesex NJ	311	671780	31.6	11.8	1637	1880	30548	79.4	26.5	3.4	5.7	24896	16725	1
62 Essex MA	498	670080	27.3	14.1	1185	2009	34312	80.2	25.9	7.5	9	22834	15301	1
63 Ventura CA	1846	669016	28.8	9.4	1168	1372	30235	79.4	23	5	7	21420	14330	4
64 Fresno CA	5963	667490	28.3	10.4	1188	1681	62004	66.2	16.9	16.8	12.6	16365	10923	4
65 Pima AZ	9187	666880	28.9	13.7	1841	2016	57051	80.5	23.3	12	3.9	15191	10131	4
66 Jefferson KY	385	664937	27	13.4	2171	3559	32419	74.1	19.3	10.9	6.3	19140	12727	3
67 Suffolk MA	59	663906	39	12.1	5674	6154	68808	75.4	27.7	14.4	8.7	23150	15369	1
68 Jefferson AL	1113	651525	26.7	14	2532	4602	55604	73.8	19.9	12.7	5.3	18624	12134	3
69 San_Mateo CA	449	649823	28.4	12.3	1814	1642	30473	84.1	31.3	4.3	4.2	28819	16791	4
70 Fulton GA	52	648951	31.6	10	3369	5757	39025	77.8	31.6	15.4	3.3	22819	14808	3
71 Jackson MO	605	633232	28.1	13.1	1695	3762	61760	79.5	20	9.8	6.5	18611	11785	2
72 Norfolk MA	400	616087	28.8	14.1	2758	1903	14830	88	34.4	3.1	7.5	26909	16578	1
73 District_of_CoDC	61	606900	33.6	12.8	3674	4262	64393	73.1	33.3	13.3	7.7	23603	14325	3
74 Oklahoma OK	709	599611	28.4	12.1	1922	3487	57045	79.1	22.6	11.3	6	17741	10638	3
75 Providence RI	413	596270	29.9	15.8	1862	2360	34627	67	18.3	8.9	9	17866	10653	1
76 El_Paso TX	1013	591610	29.5	8.1	795	1650	54002	63.7	15.2	22.4	10.8	11545	6830	3
77 Pierce WA	1676	586203	29.5	10.5	915	1226	41980	83.2	17.5	8.7	6.4	16194	9493	4
78 Multnomah OR	435	583887	28.4	13.6	2571	3009	58216	82.9	23.7	8.9	5.1	19215	11219	4
79 Travis TX	989	576407	38	7.3	1254	1392	60961	83.4	34.7	10.2	4.8	18340	10571	3
80 Montgomery OH	462	573809	28.1	12.6	1313	3068	36665	77.8	20	9.8	5.7	18410	10564	2
81 Monmouth NJ	472	553124	25.6	12.7	1300	1904	22302	82.8	28.4	3.4	5.8	27391	15151	1
82 Hudson NJ	47	553099	31.5	12.7	1036	2443	40581	64.1	19.7	12.4	9	18463	10212	1
83 Delaware PA	184	547651	27.6	15.5	1374	1588	18924	81.4	24.8	5	5.3	23658	12956	1
84 De_Kalb GA	268	545337	32.6	8.5	1035	922	56950	83.9	32.7	7.2	4.4	21005	11465	3
85 Kern CA	8142	543477	28.3	9.7	682	1194	36318	67.6	13.3	13.7	11.8	15881	80531	4
86 Bucks PA	608	541174	26.6	10.9	879	1435	16894	82.9	24.8	2.9	6.7	22548	12202	1
87 Lake IL	448	516418	28.9	8.4	1093	1499	22349	84.7	32	3.7	4.6	27378	14138	2
88 Summit OH	413	514990	26.8	13.8	1216	2226	26228	78.3	19.7	9.5	6	18583	9570	2
89 Mecklenburg NC	527	511433	32	9.4	1114	2021	57999	81.6						

Sheet1

127	Sonoma	CA	1576	388222	25.8	13.4	840	798	18556	84.4	24.5	5.2	5.7	22055	8562	4
128	Hidalgo	TX	1569	383545	26.4	10.1	311	860	26712	46.6	11.5	36.3	17.6	8899	3413	3
129	East_Baton_RL	LA	456	380105	31.5	9.2	841	1876	41592	80.5	27.5	15.1	5.4	17881	6797	3
130	Mobile	AL	1233	378643	26.7	11.8	850	1898	30409	70.1	15.5	17.5	7.2	14389	5408	3
131	Chester	PA	756	376396	27.1	10.9	594	920	9491	84.9	34.7	3	4.6	24732	9349	1
132	Volusia	FL	1106	370712	24.3	22.8	495	1345	25736	75.4	14.8	7.9	6.9	15648	5801	3
133	Stanislaus	CA	1495	370522	27.4	10.8	558	1306	25461	68.4	13	11.4	14.3	15238	5645	4
134	WestmorelandPA	PA	1023	370321	23.3	17.1	522	1306	7445	77.7	15.4	8.6	6.9	17069	6321	1
135	Santa_BarbaraCA	CA	2739	369608	32.8	12.3	875	1031	18313	80	26.6	7.4	6	21902	8095	4
136	Stark	OH	576	367585	24.9	14.4	595	1537	17466	76	14.3	8.9	6.9	16898	6211	2
137	Dane	WI	1202	367085	35.6	9.3	1603	1382	20344	88.9	34.2	4.8	3.1	20087	7374	2
138	Spokane	WA	1764	361364	27	13.2	852	1346	20042	84.4	20.6	9.8	6.2	16365	5914	4
139	Will	IL	837	357313	27.4	8.6	298	746	16432	80.4	18	4.5	7.4	18787	6713	2
140	Monterey	CA	3322	355660	32.6	9.8	515	602	17870	72.9	21.5	8.5	10.9	19465	6923	4
141	Johnson	KS	477	355054	27.5	9.4	1173	925	15238	92.9	40.5	2.5	3.3	26156	9287	2
142	Gwinnett	GA	433	352910	32.6	4.7	271	439	17119	86.1	29.6	2.9	4	19861	7009	3
143	Pulaski	AR	771	349660	28.5	11.5	1510	2785	42404	79	23.5	10.5	5.8	18225	6373	3
144	Guilford	NC	650	347420	30.4	11.9	676	1188	28212	76.1	24.8	7.3	5.4	20349	7070	3
145	Solano	CA	828	340421	29.7	8.2	481	503	21756	82.7	18.7	6	7	17268	5878	4
146	York	PA	905	339574	26.6	13.1	460	951	11292	72.8	13.9	4.3	6.2	19502	6622	1
147	Berks	PA	859	336523	26.1	15.6	567	1041	12827	70	15.1	5.2	6.7	19655	6614	1
148	Hillsborough	NH	877	336073	30	10.3	587	1050	12843	82.2	26.4	4	7.5	22581	7589	1
149	Knox	TN	509	335749	30	12.7	984	2178	22422	74.6	23.9	10.2	17.32	17382	5836	3
150	Lee	FL	804	335113	21.5	24.7	509	1202	18442	76.9	16.4	6.1	6.4	18877	6326	3
151	Luzerne	PA	891	328149	24.1	19.7	594	1495	4982	72.7	13.1	8.3	8.8	16405	5383	1
152	Mercer	NJ	226	325824	29.2	13	994	1724	20153	77.1	29.5	5.3	5.5	26026	5480	1
153	Greenville	SC	792	320167	28.2	11.9	650	1358	20504	71.6	21	7.8	4.9	17874	7423	3
154	Kane	IL	521	317471	27.8	9.3	473	1263	16721	77.7	21.4	5	7.5	21684	6884	2
155	Tulare	CA	4824	311921	26.3	10.8	358	656	19489	60.2	11.8	18	17.1	14710	4588	4
156	Washington	OR	724	311554	27.6	10.1	353	294	12630	88.2	29.8	4.8	4.4	19932	6210	4
157	Orange	NY	816	307647	28	10.4	479	986	10975	77.2	19.5	6.4	6.7	19788	6088	1
158	Waukesha	WI	556	304715	24.4	9.8	687	677	8935	88	27.1	2.2	4.1	23004	7010	2
159	Allen	IN	657	300836	27.4	11.4	552	1268	19842	81.2	19	5.6	5.9	19123	5753	2
160	Charleston	SC	917	295039	34.1	10.1	1357	1956	28190	75.5	22.4	13.8	4.9	16015	4725	3
161	Albany	NY	524	292594	30.4	14.7	1257	1246	15077	80.9	28.3	6	4.5	21003	6145	1
162	Butler	OH	467	291479	29.9	10.2	308	878	13850	76	18.7	7.6	6.6	16750	4882	2
163	Nueces	TX	836	291145	27.3	10.1	584	1406	28606	68.9	17	17.2	7.7	15124	4403	3
164	Lehigh	PA	347	291130	26.3	15.4	637	1305	12254	74.6	19.6	4.9	6.6	19785	5760	1
165	Seminole	FL	308	287339	27.9	10.3	352	352	17518	84.6	26.3	5.3	6.1	17885	5142	3
166	Richland	SC	757	285720	34.7	9.5	999	1207	24101	79.4	28	10.1	4.6	17137	4896	3
167	Hamilton	TN	543	285536	26.3	13.5	738	1573	23532	72.5	19.7	10.2	5.9	18242	5209	3
168	Washtenaw	MI	710	282937	39.5	7.5	2188	1730	19367	67.2	41.9	6.4	6	22782	6446	2
169	Lane	OR	4554	282912	27.4	13.1	497	654	16091	83	22.2	9.4	6.5	15701	4442	4
170	Ingham	MI	559	281912	27.4	8.7	729	1438	17337	83.9	29.2	11	6.9	17458	4922	2
171	Pasco	FL	745	281131	18.4	32.3	308	941	12509	66.9	9.1	7.9	8.3	13944	3920	3
172	Clackamas	OR	1868	278850	23.1	11.5	462	345	12855	85.7	4.2	23.6	4.7	19942	5661	4
173	Sarasota	FL	572	277776	18.2	32.1	631	1363	19801	81.3	21.9	4.6	5.1	24948	4930	3
174	Erie	PA	802	275572	27.5	13.8	468	1417	9936	77.5	16.2	9.5	7.1	16331	6500	1
175	Dakota	MN	570	275227	30.6	6.4	201	283	10953	90.7	27.6	3.3	4.7	21123	5814	2
176	Cumberland	NC	653	274566	37.4	6.2	291	586	25247	80.3	16.6	12.1	6.4	12923	3548	3
177	Denton	TX	889	273525	36.9	5	216	458	20372	86.8	32.3	4.5	5.6	17801	4869	2
178	Lorain	OH	493	271126	26.4	11.6	291	941	9864	75.3	12.3	9.1	9.5	15006	4630	3
179	Forsyth	NC	410	265878	29.2	12.3	1194	1659	21554	77.6	24.1	7.8	4.4	20645	5489	3
180	Rockland	NY	174	265475	25.5	10.1	931	745	7194	83.3	33	4.1	5.3	26757	7103	1
181	Adams	CO	1192	265038	29.6	7.6	439	318	19369	78.8	13	8.8	5	16116	4271	4
182	Mahoning	OH	415	264806	23.5	17.1	601	1473	13181	74.6	14	12.8	6.7	16256	4305	2
183	Collin	TX	848	264036	29.8	5.3	282	571	17625	88.3	39.1	3.9	5.9	22303	5899	3
184	Utah	UT	1998	263590	33.9	7	291	544	10605	87.9	26.2	10.6	4.3	11467	3023	4
185	St_Clair	IL	664	262852	26.8	12.7	329	1088	14563	72.6	14.7	13.9	8.4	16190	4256	2
186	Escambia	FL	664	262798	29.2	11.9	522	1584	14380	76.2	18.2	13.3	5.8	15392	4045	3
187	Norfolk_City	VA	54	261229	41.7	10.5	1101	1471	25194	72.7	16.8	15.1	6.4	16412	4287	3
188	Cameron	TX	906	260120	25.9	10.6	270	825	18842	50	12	33.7	12.5	9728	2530	3
189	Dutchess	NY	802	259462	29	11.4	535	741	9087	79.8	24.8	3.6	5.1	22173	5753	1
190	New_London	CT	666	254957	31.2	11.9	486	515	7807	80.9	21.8	4.7	6.8	20259	5165	1
191	Washoe	NV	6343	254667	29.5	10.3	603	990	18831	4	6.3	5	5	21327	5431	4
192	Hinds	MS	869	254441	29.5	11.2	1076	2118	28841	75.2	26.4	16.7	6.3	16215	4126	3
193	Winnebago	IL	514	252913	26.5	12.7	521	910	19674	76.3	16.7	7.7	7.9	18376	4648	2
194	Oneida	NY	1213	250836	27.7	15.5	437	905	9234	75.1	16.7	8.8	6.7	16477	4133	1
195	Madison	IL	725	249238	26.2	13.9	275	1120	10666	75.8	14.4	5.5	7.9	17980	4481	2
196	Caddo	LA	882	248253	25.2	13.3	898	1868	22051	73.4	18.2	18.1	7.1	16337	4056	3
197	Northampton	PA	374	247105	26.9	15	459	933	6452	73.1	16.7	4.9	6.8	18336	4531	1
198	St_Joseph	IN	457	247052	28.2	14.1	417	927	10637	76.1	19.2	7.1	5.7	17211	4252	2
199	Rockingham	NH	695	245845	29	9.2	343	514	7295	86.2	25.9	3.2	8	21770	5352	1
200	Cumberland	ME	836	243135	29.1	13	732	1104	13816	85	27.6	5.7	5.7	21362	5194	1
201	Somerset	NJ	305	240279	28.6	10.8	783	374	8308	86.3	38.3	1.4	4.1	33180	4712	1
202	Jefferson	TX	904	239397	25.8	14	449	1724	21677	74.4	15.5	15.5	6.7	17418	7970	3
203	Madison	AL	805	238912	31.4	8.9	399	933	6635	80.2	30.1	8.4	5.2	18990	4537	3
204	Clark	WA	628	238053	25.3	10.6	256	299	10706	83.9	16.8	7	6.6	16790	3997	4
205	Dauphin	PA	525	237813	26.8	14.3	824	1425	11563	77.6	18.6	7.2	5.5	18348	4363	1
206	Marin	CA	520	230096	24.7	12.3	1001	488	9460	91.9	44	3	4	37541	4838	4
207	Gloucester	NJ	325	230082	28	10.8	199	339	9746	77.5	18.1	4.7	7.1	18523	4262	1
208	Santa_Cruz	CA	446	229734	29.9	11.3	429	390	13707	81.9	29.7	6.2	8	22025	5060	4
209	Marion	OR	1185	228483	25.7	14.3	376	498	14825	78.7	17.5	9.4	5.7	16022	3661	4
210	Trumbull	OH	616	227813	23.9	14.4	225</									

253 Peoria	IL	620	182827	26	14.2	581	1219	12483	77.9	19.5	11.3	7.2	18342	3353	2	
254 Montgomery	TX	1044	182201	25.5	8.6	125	340	9469	75.5	19.4	9.4	5.5	17084	3113	3	
255 Harford	MD	440	182132	28.8	8.3	247	333	6735	81.6	21.5	4.1	6.7	20941	3814	3	
256 Butte	CA	1640	182120	28.2	17.3	327	625	8939	77.6	19.5	12.2	9.4	15051	2741	4	
257 Clayton	GA	143	182052	32.4	5.8	191	346	15419	77.2	14.7	7.3	5.8	16171	2944	3	
258 Durham	NC	291	181835	33.7	10.7	144	1496	15477	78.9	33.4	4.7	3.6	19238	3498	3	
259 Alachua	FL	871	181596	40.1	9.3	1180	1018	18218	82.7	34.6	14.4	4.2	16058	2816	3	
260 Saratoga	NY	812	181276	28.4	10.3	215	221	5281	83	25.2	8.8	5.9	18857	3418	1	
261 Muscogee	GA	216	179278	30.6	10.8	360	1168	11454	71.5	16.6	14.9	4.6	15505	2780	3	
262 Merced	CA	1929	178403	28.2	9.2	185	337	8587	63.1	12	15.4	14.6	13961	2491	4	
263 Sangamon	IL	868	178386	25.8	13.8	600	1330	11929	81.8	22.4	7.2	4.8	19601	3497	2	
264 Gaston	NC	357	175093	27.3	12.1	142	368	11865	60.9	10.8	8.2	6.2	16319	2857	3	
265 Racine	WI	333	175034	26	12	234	532	11110	76.4	16.5	7.9	6.5	18426	3225	2	
266 Buncombe	NC	656	174821	24.8	16.1	469	725	9512	74.5	19.1	8.2	4.9	16934	2960	3	
267 Cleveland	OK	536	174253	33.9	6.7	217	319	12194	83.9	25.9	7	4.8	14443	2517	3	
268 Litchfield	CT	920	174092	25.4	14.1	278	411	3593	80.9	25	2.4	7	25161	4380	1	
269 Champaign	IL	997	173025	41.6	8.8	382	805	11508	87.5	34.1	8	4.5	16957	2934	2	
270 Placer	CA	1404	172796	23.5	12	329	322	8904	64.4	85.1	22.7	5.3	6.8	20168	3485	4
271 Jefferson	MO	657	171380	28.5	8.3	61	230	3128	71.6	9	6	8.7	15896	2724	2	
272 Arlington_CouVA		26	170936	37.6	11.3	615	781	12526	87.5	52.3	4.3	3.6	30242	5169	3	
273 Newport_NewVA		68	170045	33.9	9.3	354	836	11776	79.3	18.4	12.2	6.5	15327	2606	3	
274 Calcasieu	LA	1071	168134	26.6	10.9	248	845	6399	70.3	14.7	15.5	7.8	14968	2517	3	
275 Lexington	SC	701	167611	27.8	8.9	145	259	9814	77.3	21	6.3	4.1	18126	3038	3	
276 Harrison	MS	581	165365	30	10.8	313	764	7043	74.7	16.3	15.4	6.7	13691	2264	3	
277 Ulster	NY	1127	165304	27.8	13	258	413	4701	76.6	21.6	5.6	5.9	18824	3112	1	
278 Vanderburgh	IN	235	165058	27	15.7	411	1376	8405	75.2	16	9.1	5.7	18093	2886	2	
279 Lafayette	LA	270	164762	31.1	8.3	361	1018	10599	73.3	22.5	16.2	5	16868	2799	3	
280 York	ME	991	164587	26.4	12.6	172	404	6027	79.5	19	4.7	6.9	17908	2947	1	
281 Cambria	PA	688	163029	23	18.7	301	892	3187	71.2	10.8	11.2	9.2	14473	2360	1	
282 Wyandotte	KS	151	161993	27.4	13	494	1019	18902	69.9	10.3	13.9	7.4	14134	2290	2	
283 Berrien	MI	571	161378	25.2	13.7	199	688	12229	74.7	16.7	11.6	9.1	16232	2619	2	
284 Thurston	WA	727	161238	25.3	11.7	283	500	7882	86.5	24.7	7.1	5.9	17312	2791	4	
285 Kent	RI	170	161135	26.2	15.1	264	359	7302	76.8	20.5	3.7	8.6	20086	3237	1	
286 Shawnee	KS	550	160976	26.2	13.1	451	661	13845	84.4	22.3	7.4	4.7	19558	3148	2	
287 Muskogee	MI	509	158983	25.8	13	182	660	12181	74.2	11.1	12.4	12	14767	2348	2	
288 Weber	UT	576	158330	26	11.2	166	573	9191	75.5	18	7.8	5.7	15301	2423	4	
289 Elkhart	IN	464	156196	26.8	11.2	164	478	7573	72.8	14.2	5.3	6.4	16770	2615	2	
290 Wenselslaer	NY	654	154439	29.8	13.2	213	616	5287	77.7	19.5	6.6	6.3	17774	2745	1	
291 Clay	MO	397	153411	28.3	10.4	108	639	11085	84.7	20	4.2	5.7	18395	2622	2	
292 Schuylkill	PA	779	152585	22.9	20	147	634	2119	68.4	8.1	7.7	9.7	15853	2419	1	
293 Lake	FL	953	152104	19	27.5	167	664	7099	70.6	12.7	7.9	8.9	17496	2661	3	
294 Collier	FL	2026	152099	22.4	22.8	282	431	9426	79	22.3	6.4	7.5	25589	3892	3	
295 Butler	PA	789	152013	27	13.5	127	261	3420	78.6	15.6	7.1	6.8	17251	2622	1	
296 Chesapeake_VA		341	151976	28.6	8.4	212	210	8427	77.1	16.9	7	6.1	16924	2572	3	
297 Smith	TX	929	151309	26.2	13.7	349	795	11712	75.7	19.8	12.6	6.6	17511	2650	3	
298 Tuscaloosa	AL	1325	150522	33.3	11.4	299	731	12377	69.6	20	13.5	5.4	15113	2275	3	
299 Frederick	MD	663	150208	28.7	9.4	172	241	4939	80.4	22	3.5	6	19954	2997	3	
300 Clermont	OH	452	150187	28.1	8.7	82	151	5114	74.8	14.5	7.4	6	16231	2438	2	
301 St_Lucie	FL	573	150171	22.9	21	176	425	9842	71.7	13.1	8.5	13.8	14137	2123	3	
302 Bibb	GA	250	149967	27.5	12.9	438	1010	12701	68.2	17	15.9	4.1	17548	2632	3	
303 Onslow	NC	767	149838	49.7	4.4	104	133	7505	83	13.4	9.8	5.6	10190	1527	3	
304 Jackson	MI	707	148756	27.1	12.3	127	573	8630	77.7	12.9	9.4	9.9	15750	2359	2	
305 Schenectady	NY	206	149285	26.1	16.5	403	721	6364	80.7	23	5.7	5.6	20679	3087	1	
306 Rock_Island	IL	427	148723	24.9	15	209	769	7154	77.4	15	10.2	7.5	17818	2650	2	
307 Clark	OH	400	147548	25.3	13.8	173	463	10131	73.4	12.2	10.4	6.5	16676	2461	2	
308 Shasta	CA	3786	147036	22.8	14.1	267	468	7336	78.4	13.7	11	10.3	16277	2393	4	
309 Penobscot	ME	3396	146601	30	11.5	268	598	4749	79.1	17.7	9.5	8	15521	2275	1	
310 Hampshire	MA	529	146568	38.2	11.6	348	236	2547	83	31.9	5.7	7.7	17853	2617	1	
311 Jackson	OR	2785	146389	22.1	16.2	263	522	7170	80.1	17.6	9.7	7.6	15582	2281	4	
312 Washington	MN	392	145896	26.5	6.6	113	92	5365	90	26.2	3.6	4.6	20682	3017	2	
313 St_Claire	MI	725	145607	25.9	12.3	143	431	6568	74.8	10.7	9.2	11.8	17480	2545	2	
314 Fayette	PA	790	145351	22.9	18	124	409	3612	67.8	9.3	17.6	9.4	14051	2042	1	
315 Anderson	SC	718	145196	25.5	13.6	199	456	7525	64	12.9	8.6	6.6	14205	2063	3	
316 St_Tammany	LA	854	144508	24.2	8.9	282	512	4447	76.9	23.1	11.1	6.2	17129	2475	3	
317 Horry	SC	1134	144053	28.2	12.7	175	505	12459	74.3	16	11.6	7.7	14693	2117	3	
318 Okaloosa	FL	936	143776	30.8	9.3	178	482	5153	83.8	21	7.8	6.7	15803	2272	3	
319 Sullivan	TN	413	143596	24.6	14.3	377	982	6236	66.8	15.6	10.4	4.4	15747	2261	3	
320 Middlesex	CT	369	143196	28.4	13.1	340	235	3409	82.6	28.2	2.6	5.8	24132	3456	1	
321 Portage	OH	492	142855	33.6	9.4	101	285	2769	79.3	17.5	7.6	6.8	16031	2286	2	
322 Ouachita	LA	611	142191	28.3	11.2	268	1043	10605	71.6	18.9	18.6	6.4	13869	1872	6	
323 Kenton	KY	163	142031	28.3	11.5	263	733	6925	74.4	17	8.2	5.4	16935	2405	3	
324 Chautauqua	NY	1062	141895	25.7	15.7	164	653	5178	74.4	14.2	9.9	7.7	15197	2156	1	
325 Yolo	CA	1012	141092	36.5	9.6	339	168	10650	79.1	30.3	9.8	7.2	19727	2783	4	
326 Outagamie	WI	640	140510	28.2	11.1	228	511	4860	81.5	16.7	4.6	4.9	17182	2414	2	
327 Winnebago	WI	439	140320	30.3	12.8	242	528	6170	80.6	18.2	5.3	4.8	17645	2476	2	
328 Williamson	TX	1124	139551	29.1	7.6	88	185	5724	81.4	24.6	7.6	3.8	14934	2084	3	
329 Rock	WI	721	139510	26.2	12.6	171	491	7643	78.2	13.3	7.8	9.7	16742	2336	2	
330 Berkshire	MA	931	139352	26	16.9	375	598	3862	77.9	20.9	6.3	10.1	20068	2797	1	
331 Cumberland	NJ	489	138053	26.7	13.5	181	534	9071	63.4	10.8	10.2	10.7	16819	2322	1	
332 Greene	OH	415	136731	28.5	9.8	134	210	5221	82.4	26	7.1	5.2	18161	2483	2	
333 Calhoun	MI	709	135982	25.1	13.3	172	566	9810	76.8	13.8	11	8.5	15944	2168	2	
334 Dona_Ana	NM	3807	135510	31.4	8.8	171	240	8850	70.4	21.9	20.7	7.4	11379	1542	4	
335 Hampton_CityVA		52	133793	33	9.6	163	251	8376	79.7	19.1	8.8	6.8	14743	1973	3	
336 Monroe	MI	551	133600	26.1	8.4	83	182	6726	74.1	10.5	6.9	11.1	17278	2308	2	
337 Webb	TX	3357	133239	28.5	7.9	107	382	12202	47.8	11.1	33.1	9.8	8973	1196	3	
338 Weld	CO	3993	131821	29.6	10.2	172	281	7901	74.9	18.4	10.6	6.5	15874	2093	4	
339 Chittenden	VT	539	131761	35.8	8.1	696	573	4739	86.7	34	5	4.4	19940	2627	1	

Sheet1

379 Macon	IL	581	117206	24.1	14.5	171	725	6103	76.2	14.8	9.8	8.9	18021	2112	2
380 Pinal	AZ	5370	116379	24.4	13.7	61	309	6275	65.5	8.2	18.7	7.9	11396	1326	4
381 Calhoun	AL	609	116034	28.8	12.4	133	486	4901	67.4	14.2	11.7	7.3	13776	1598	3
382 Kennebec	ME	868	115904	26.1	13.4	241	497	4184	78.9	18.1	7.3	6.8	17131	1986	1
383 Livingston	MI	568	115645	25.3	8.2	68	93	3760	85.6	19.6	3	8.1	21153	2446	2
384 Marathon	WI	1545	115400	25.9	12.7	172	254	3655	75.9	13.5	5.5	5.9	16305	1882	2
385 Jackson	MS	727	115243	25.9	9.4	170	346	4777	74.4	14.4	14	7.1	13475	1553	3
386 Florence	SC	799	114344	26.2	11.2	211	731	8421	64.3	14.8	15.9	5.9	14961	1711	3
387 Lebanon	PA	362	113744	25.3	15	162	196	2919	70	11.8	5.2	5.9	16500	1877	1
388 Yellowstone	MT	2635	113419	25.6	12.4	262	554	3879	83.7	21.5	9	5.3	17272	1959	4
389 Washington	AR	950	113409	32	11.2	208	651	6122	73.2	20	9.8	3.7	14736	1671	3
390 Wood	OH	617	113269	34.5	10.2	128	124	3759	83.8	21.9	5	6.6	17522	1985	2
391 Benton	WA	1703	112560	25.1	10.1	142	278	6249	83.9	23.3	8.9	6.8	17332	1951	4
392 Boone	MO	685	112379	40.9	8.4	746	1023	5456	84.8	36.5	9.3	3.2	17175	1930	2
393 St._Lawrence	NY	2686	111974	31.2	12.1	132	378	3851	73.1	15.1	12.8	9.8	12704	1423	1
394 Bay	MI	444	111723	25.4	13.4	101	415	4849	74	11	10.2	9.2	16499	1843	2
395 Comanche	OK	1069	111486	34.5	8.7	127	347	5979	81.1	18.4	13.3	6.6	13228	1475	3
396 Alexandria_CIVA		15	111183	38.3	10.3	652	662	8537	86.9	48.5	4.7	4.6	31699	3524	3
397 Kent	DE	591	110993	29.7	10.3	123	193	5846	73.1	15	8.7	7.6	14946	1659	1
398 Charlotte	FL	694	110975	16.6	33.8	183	632	3741	75.7	13.4	5.2	7.2	16362	1816	3
399 Jefferson	NY	1272	110943	32.7	10.9	124	336	3064	76.4	13.6	9.5	11	15205	1687	1
400 Napa	CA	754	110765	24.5	16.5	345	1019	5056	80.7	22.3	4.6	5.9	22668	2511	4
401 Rowan	NC	511	110605	26	15.2	114	244	5233	66	11.7	6.8	5.4	15691	1736	3
402 Washington	RI	333	110006	31	12.3	162	241	3838	82.8	29.1	3.7	6.7	19449	2140	1
403 Allen	OH	405	109755	26.2	13.4	168	560	4734	76.1	11.4	10.4	7.3	16542	1816	2
404 Imperial	CA	4175	109303	25.5	10.2	82	221	8042	53.2	9.7	20.8	21.3	14523	1587	4
405 Monroe	IN	394	108978	45.8	8.6	172	285	1657	82.1	32.9	9.5	3.7	14266	1555	2
406 Hamilton	IN	398	108936	25.1	8.2	257	122	1699	88.7	36.2	2.6	2.8	25681	2798	2
407 Columbiana	OH	533	108276	23.4	14.9	80	485	898	71.8	8.5	13	6.7	12597	1364	2
408 Alamance	NC	431	108213	27.3	14.8	132	340	4152	67.9	14.6	6	4.5	17306	1873	3
409 Pitt	NC	652	107924	35.4	9.9	496	583	4603	71	21.9	15.5	5.5	15852	1711	3
410 Hunterdon	NJ	430	107776	25.6	9.5	184	182	2068	85.9	34.6	1.8	4.1	30255	3261	1
411 Osceola	FL	1322	107728	27.1	13.9	98	291	9665	73.7	11.2	6.9	6.7	16451	1772	3
412 Yavapai	AZ	8124	107714	18.3	23.8	114	159	3952	78.9	17.7	9.8	5.4	13681	1474	4
413 La_Porte	IN	598	107066	26	13.1	149	519	6021	73.9	11.7	7.8	5.8	16655	1783	2
414 La_Salle	IL	1135	106913	23.8	17.2	104	504	2962	73.1	10.5	8.7	9.6	16119	1723	2
415 Yuma	AZ	5514	106895	27.4	13.8	118	137	5414	64.9	12.7	15.4	17.8	11490	1228	4
416 Midland	TX	900	106611	26.8	9	139	333	7546	76.8	26.4	11.5	5.4	19345	2062	3
417 Randolph	NC	788	106546	27.1	12.2	69	145	2940	62	9.1	6.5	4.8	14721	1568	3
418 Olmsted	MN	653	106470	29.3	10	1814	1437	4310	88	29.5	4.5	3.3	20515	2184	2
419 Vigo	IN	403	106107	30.2	15.1	179	576	3435	76	18.1	10.7	5	15036	1595	2
420 Clay	FL	601	105986	26.3	8.5	164	277	4560	81.2	17.9	5.4	5.9	16029	1699	3
421 Androscoggin	ME	470	105259	27.9	13.4	198	527	4020	71.8	12.6	8.7	9.7	16154	1700	1
422 Robeson	NC	949	105179	26.7	10.7	83	281	4318	57	11	20.7	8.9	10849	1141	3
423 Gregg	TX	274	104948	26.4	13.3	166	420	9181	75.8	17.7	13.4	8.2	16775	1761	3
424 Wayne	NC	553	104666	29.7	10.2	113	263	4682	71.2	12.7	11.9	7.3	13350	1397	3
425 Strafford	NH	369	104233	34.8	10.7	139	237	3651	79.8	21.7	5.1	6.5	17182	1791	1
426 Sheboygan	WI	514	103877	25.4	14.6	114	421	4433	77.4	13.8	4.5	5.6	18061	1876	2
427 Fairfield	OH	506	103461	25.2	11.3	86	195	625	78.8	15.5	6.8	6.3	16342	1691	2
428 Sumner	TN	529	103281	25.5	10.2	96	259	3285	70.6	14.4	7	6.8	16514	1706	3
429 Cass	ND	1766	102874	34.4	9.8	343	643	3401	87.1	26.5	7.3	2.5	16275	1674	2
430 Sumter	SC	666	102637	31.6	9.4	88	214	7138	69.8	15	16.9	9.4	11803	1211	3
431 Sarpy	NE	241	102583	30.4	4.8	39	160	2689	91	25.4	3.5	2.6	16137	1655	2
432 Windham	CT	513	102525	28.5	12.5	123	254	1397	71.1	16.8	6	9.2	18070	1853	1
433 Kings	CA	1390	101469	33.7	7.7	82	180	4449	65.6	9	15	12.8	13907	1411	4
434 Wayne	OH	555	101461	26.3	11.6	84	155	2377	73.6	13.9	8.4	5.9	16464	1670	2
435 Charles	MD	461	101154	29.9	6.5	67	104	5279	81	16.2	3.7	4.9	19317	1954	3
436 Hernando	FL	478	101115	16.4	30.7	98	290	4414	70.5	9.7	7.9	8.2	13919	1407	3
437 Martin	FL	556	100900	20.4	27.5	193	277	5081	79.7	20.3	5	9.8	27125	2737	3
438 Montgomery	TN	539	100498	35.7	7.9	87	188	6537	77.9	16.5	10.8	8	13169	1323	3
439 Maui	HI	1159	100374	26.2	11.3	192	182	7130	77	17.8	5.7	3.2	18504	1857	4
440 Morgan	AL	582	100043	26.3	11.7	122	464	4693	69.4	15.5	9.4	7.1	16458	1647	3